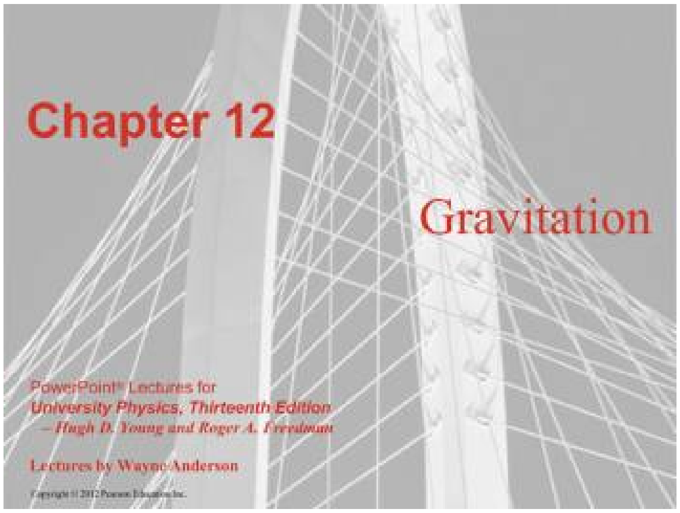


Chapter 12

Gravitation

PowerPoint® Lectures for
University Physics, Thirteenth Edition
– Hugh D. Young and Roger A. Freedman

Lectures by Wayne Anderson

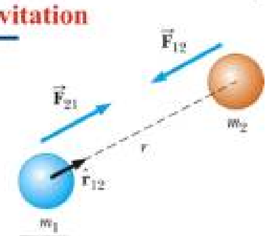
Goals for Chapter 12

- To calculate the gravitational forces that bodies exert on each other
- To relate weight to the gravitational force
- To use the generalized expression for gravitational potential energy
- To study the characteristics of circular orbits
- To investigate the laws governing planetary motion
- To look at the characteristics of black holes

Newton's Law of Universal Gravitation

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the distance between them

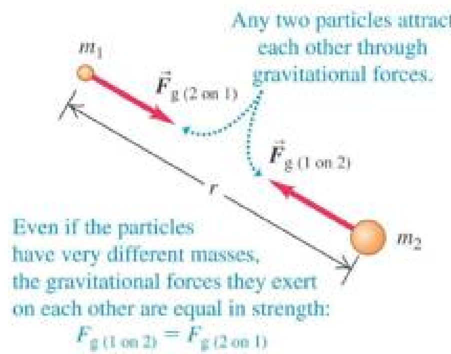
$$F_g = G\frac{m_1 m_2}{r^2}$$



G is the ***universal gravitational constant*** and equals 6.673 x 10^{-11} Nm^2 / kg^2

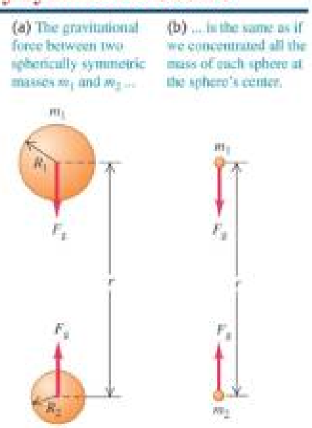
$$G = 6.673 \times 10^{-11}\ \text{Nm}^2 / \text{kg}^2$$

Newton's law of gravitation



Gravitation and spherically symmetric bodies

- The gravitational interaction of bodies having *spherically symmetric* mass distributions is the same as if all their mass were concentrated at their centers

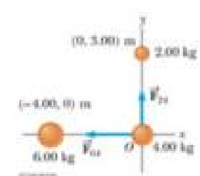


Example

Three uniform spheres of mass 2 kg, 4 kg, and 6 kg are placed at the corners of right triangle. Calculate the net gravitational force on the 4 kg object assuming the spheres are isolated from the rest of the Universe. $G = 6.673 \times 10^{-11}$ Nm^2 / kg^2

$F_g = G\frac{m_1 m_2}{r^2}$

Apply superposition rule



g Above the Earth's Surface

If an object is some distance h above the Earth's surface, r becomes $R_E + h$

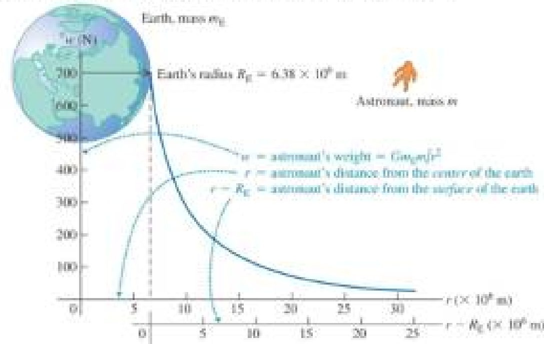
$$g = \frac{GM_E}{(R_E + h)^2}$$

This shows that g decreases with increasing altitude

As $r \to \infty$, the weight of the object approaches zero

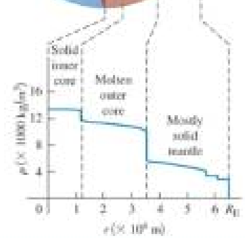
Weight

- The *weight* of a body decreases with its distance from the earth's center, as shown in Figure 13.8 below.



Interior of the earth

- The earth is approximately spherically symmetric, but it is *not* uniform throughout its volume, as shown in at the right.



Example

A robotic lender with an earth weight of *3430 N* is sent to Mars which has a radius $R_M = 3.4 \times 10^6 m$ and mass $m_m = 6.42 \times 10^{23} kg$. Find the weight F_g of the lander on the Martian surface.

Variation of *g* with Height

TABLE 13.1

Free-Fall Acceleration *g* at Various Altitudes Above the Earth's Surface

Altitude h (km)	g (m/s²)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13
∞	0

Gravitational Potential Energy

The gravitational force is conservative

The change in gravitational potential energy of a system associated with a given displacement of a member of the system is defined as the negative of the work done by the gravitational force on that member during the displacement

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\,dr$$

Gravitational Potential Energy, cont

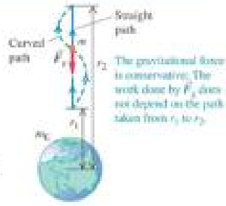
As a particle moves directly away from the center of the earth from r_1 to r_2 the work done by gravitational force is

$$W_g = \int_{r_i}^{r_f} F(r)\,dr \qquad F(r) = -G\frac{M_E m}{r^2}$$

$$W_{\text{grav}} = -Gm_E m\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{Gm_E m}{r_2} - \frac{Gm_E m}{r_1}$$

$$U_f - U_i = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Choose $U_i = 0$ where $r_i = \infty$ $\qquad U(r) = -\frac{GM_E m}{r}$



- This is valid only for $r \geq R_E$ and not valid for $r < R_E$
- U is negative because of the choice of U_i

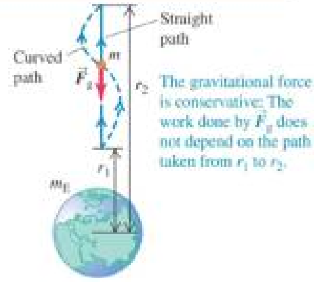
Gravitational potential energy

- The *gravitational potential energy* of a system consisting of a particle of mass *m* and the earth is

$$U(r) = -\frac{GM_E m}{r}$$

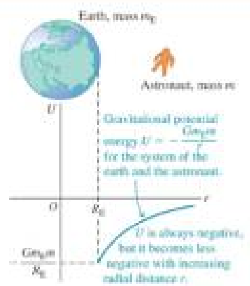
$U_i = 0$ for $r_i = \infty$ and r is a distance measured from the center of the earth

- This is valid only for $r \geq R_E$ and not valid for $r < R_E$
- U is negative because of the choice of U_i



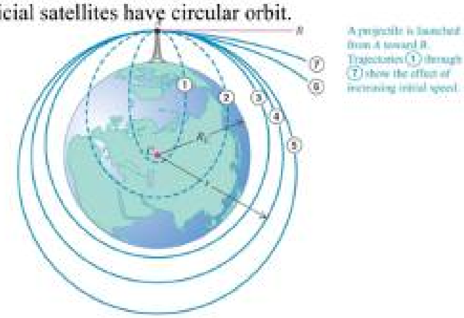
Gravitational potential energy depends on distance

- The gravitational potential energy of the earth-astronaut system *increases* (becomes less negative) as the astronaut moves away from the earth.



The motion of satellites

- The trajectory of a projectile fired from *A* toward *B* depends on its initial speed. If it is fired fast enough, it goes into a *closed elliptical orbit* (trajectories 3, 4, and 5). Orbital 4 is a circle. Most artificial satellites have circular orbit.



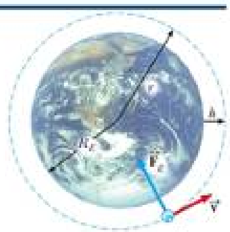
Motion of satellites

For satellite in circular motion about the earth

$$G\frac{M_E m}{r^2} = \frac{m v^2}{r}$$

and

$$v_{\text{sat}} = \sqrt{\frac{GM_E}{R}} \qquad T = \frac{2\pi r}{v}$$



where T is a period of satellite. For the satellite:

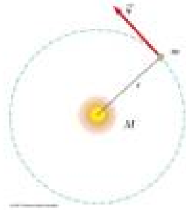
$$g = G\frac{M_E}{r^2} = \frac{V^2}{r} = a_c$$

Circular satellite orbits

- For a circular orbit, the speed of a satellite is just right to keep its distance from the center of the earth constant.
- Astronauts inside the satellite in orbit are in a state of *apparent weightlessness* because inside the satellite $N\text{-}mg = \text{-}ma_c$ $\quad g = a_c \quad N = 0$

Total Energy of Satellite

Total energy of a satellite in a circular motion is $E = K + U$



$$E = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

$$G\frac{M_E m}{r^2} = \frac{mv^2}{r}$$

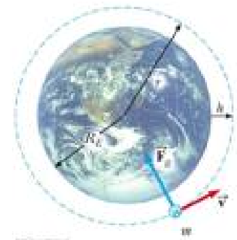
$$\frac{1}{2}mv^2 = \frac{GM_E m}{2r}$$

K = U/2 and E = K+U = U/2 –U = -U/2

$$E = -\frac{GMm}{2r}$$

Example , Geosynchronous Satellite

Find the altitude and the velocity of geosynchronous satellite.
A geosynchronous satellite appears to remain over the same point on the Earth. The period of geosynchronous satellite is 24 h



$M_e = 5.98\times10^{24}$ kg, $R_e = 6.37\times10^{6}$m

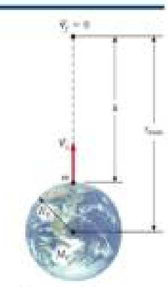
Example

You wish to put a 1000 kg satellite into a circular orbit 300 km above the earth's surface.
(a) What speed, period and radial acceleration will it have?
(b) How much work must be done to the satellite to put it in orbit?

Escape Speed from Earth

An object of mass m is projected upward from the Earth's surface with an initial speed, v_i

Using energy considerations the minimum value of the initial speed (escape speed) needed to allow the object to move infinitely far away from the Earth can be found:



$$U_i + K_i = 0+0$$

$$-\frac{GM_E m}{r} + \frac{1}{2}mv^2 = 0 + 0$$

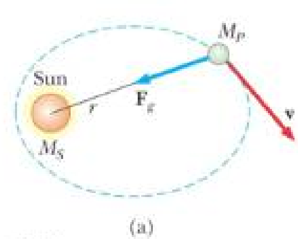
$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

Kepler's laws and planetary motion

- Each planet moves in an elliptical orbit with the sun at one focus.
- A line from the sun to a given planet sweeps out equal areas in equal times (see Figure 13.19 at the right).
- The periods of the planets are proportional to the $^3/_2$ powers of the major axis lengths of their orbits.

Kepler's Second Law

- Is a consequence of conservation of angular momentum
- The force produces no torque, so angular momentum is conserved
- $\mathbf{L} = \mathbf{r} \times \mathbf{p} = M_P\,\mathbf{r} \times \mathbf{v}$



(a)

Kepler's Third Law, cont

- This can be extended to an elliptical orbit
- Replace r with a
  - Remember a is the semimajor axis

$$T^2 = \left(\frac{4\pi^2}{GM_{Sun}}\right)a^3 = K_S a^3$$

- K_S is independent of the mass of the planet, and so is valid for any planet

Planetary Data

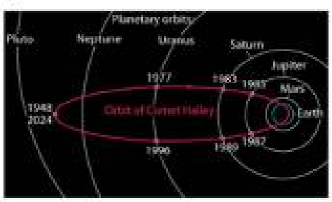
Body	Mass (kg)	Mean Radius (m)	Period (s)	Mean Dist. From Sun (m)	$T^2/R^3 = K_s$ X10-19 (s^2/m^3)
Mercury	3.18×10^{23}	2.43×10^{6}	7.6×10^{6}	5.79×10^{10}	2.97
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}	2.99
Earth	5.98×10^{24}	6.37×10^{6}	3.16×10^{7}	1.5×10^{11}	2.97
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}	2.98
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}	2.97
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}	2.99
Moon	7.36×10^{22}	1.74×10^{6}	----------	-----------	----------
Sun	1.99×10^{30}	6.96×10^{8}	-----------	-----------	-----------

Some orbital examples

(a) The orbit of Comet Halley

(b) Comet Halley as it appeared in 1986

(a)



(b)

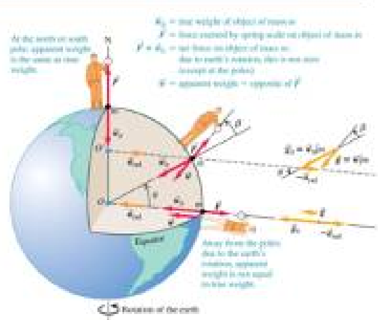
(a) The orbit of Comet Halley. (b) Comet Halley as it appeared in 1986. At the heart of the comet is an icy body, called the nucleus, that is about 10 km across. When the comet's orbit carries it close to the sun, the heat of sunlight causes the nucleus to partially evaporate. The evaporated material forms the tail, which can be tens of millions of kilometers long.

Example

Comet Halley moves in an elongated elliptical orbit around the sun (Fig. 13.20). Its distances from the sun at perihelion and aphelion are 8.75×10^7 km and 5.26×10^9 km, respectively. Find the orbital semi-major axis, eccentricity, and period.

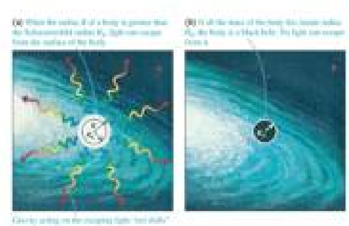
Apparent weight and the earth's rotation

- The *true weight* of an object is equal to the earth's gravitational attraction on it.
- The *apparent weight* of an object, as measured by the spring scale in Figure 13.25 at the right, is less than the true weight due to the earth's rotation.



Black holes

- If a spherical nonrotating body has radius less than the *Schwarzschild radius*, nothing can escape from it. Such a body is a *black hole*. (See Figure 13.26 below.)
- The Schwarzschild radius is $R_S = 2GM/c^2$.
- The *event horizon* is the surface of the sphere of radius R_S surrounding a black hole.



Detecting black holes

- We can detect black holes by looking for x rays emitted from their *accretion disks*. (See Figure 13.27 below.)