Chapter 10

Dynamics of Rotational Motion

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Goals for Chapter 10

- To learn what is meant by torque
- To see how torque affects rotational motion
- To analyze the motion of a body that rotates as it moves through space
- To use work and power to solve problems for rotating bodies
- To study angular momentum and how it changes with time
- To learn why a gyroscope precesses

Torque

Torque τ is the tendency of a force to rotate an object about some axis

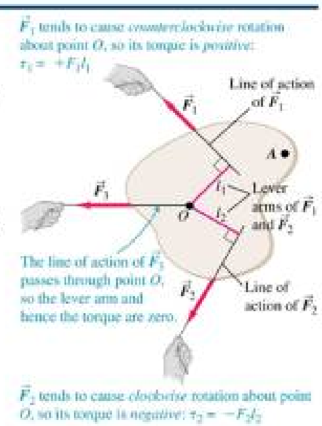
- Torque is a vector. Its magnitude is

$$\tau = r\,F\sin\theta = F\,d$$

  - F is the force
  - θ is the angle between force and r vector
  - d is the *moment arm* (or lever arm) of the force

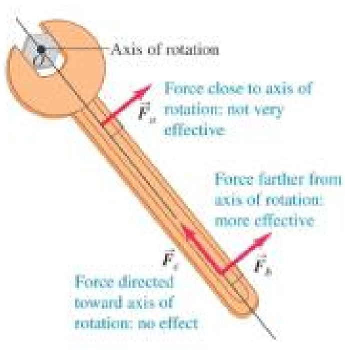
Torque

- The *line of action* of a force is the line along which the force vector lies.
- The *lever arm* (or *moment arm*) for a force is the perpendicular distance from O to the line of action of the force (see figure).
- The torque of a force with respect to O is the product of the force and its lever arm.



Loosen a bolt

- Which of the three equal-magnitude forces in the figure is most likely to loosen the bolt?

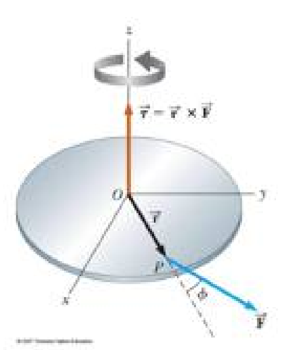


The Vector Product and Torque

The torque vector lies in a direction perpendicular to the plane formed by the position vector and the force vector

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The torque is the vector (or cross) product of the position vector and the force vector



Torque Vector Example 2

Given the force and location

$\vec{F} = (2.00\hat{i} + 3.00\hat{j})\ N$

$\vec{r} = (4.00\hat{i} + 5.00\hat{j})\ m$

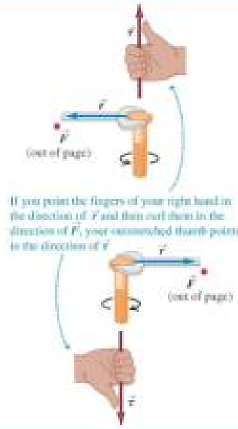
Find the torque produced

$$\vec{\tau} = \vec{r} \times \vec{F} = [(4.00\hat{i} + 5.00\hat{j})N] \times [(2.00\hat{i} + 3.00\hat{j})m]$$

$$= [(4.00)(3.00) - (5.00)(2.00)]\hat{k} = 2.0\hat{k}\ N \cdot m$$

Torque as a vector

- Torque can be expressed as a vector using the vector product.
- Figure 10.4 at the right shows how to find the direction of torque using a right hand rule.

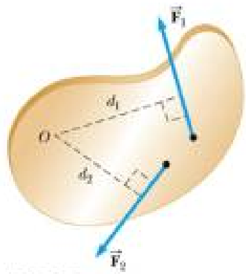


Net Torque

The force $\vec{F}_1$ will tend to cause a counterclockwise rotation about *O*

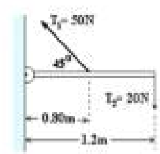
The force $\vec{F}_2$ will tend to cause a clockwise rotation about *O*

$\tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$



Net torque

What is the magnitude and direction of the net torque produced by two tension forces T_1 and T_2 about the hinge?



Torque and Angular Acceleration

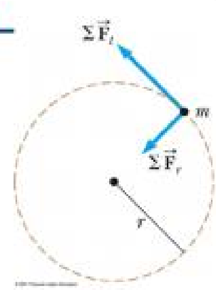
Consider a particle of mass *m* rotating in a circle of radius *r* under the influence of tangential force $\vec{F}_t$

The tangential force provides a tangential acceleration:

- $F_t = ma_t$

The magnitude of the torque produced by the tangential force F_t around the center of the circle is

- $\tau = F_t\, r = (ma_t)\, r$



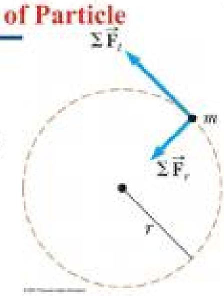
Torque and Angular Acceleration of Particle

The tangential acceleration is related to the angular acceleration

- $\tau = (ma_t)\, r = (mr\alpha)\, r = (mr^2)\, \alpha$

Since mr^2 is the moment of inertia of the particle,

$$\tau = I\alpha$$



In general the Second Newton's Law for rotation can be expressed as

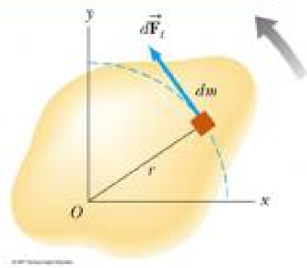
$$\tau = I\alpha$$

Torque and Angular Acceleration, Extended

Consider the object consists of an infinite number of mass elements *dm* of infinitesimal size

Each mass element rotates in a circle about the origin, *O*

Each mass element has a tangential acceleration



Torque and Angular Acceleration, Extended cont.

From Newton's Second Law

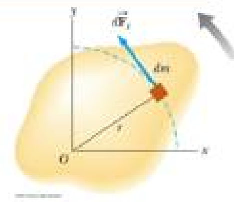
- $dF_t = (dm)\, a_t$

The torque associated with the force and using the angular acceleration gives

- $d\tau = r\, dF_t = a_t r\, dm = \alpha r^2\, dm$

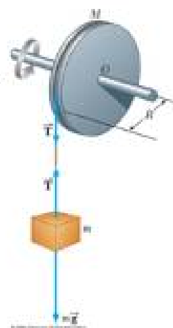
Finding the net torque

- $\sum \tau = \int \alpha r^2 dm = \alpha \int r^2 dm$
- This becomes $\tau = I\alpha$



Example

A uniform disk with radius 20 cm and mass 2.5 kg is mounted on a frictionless axle. A 1.2 kg block hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk and the tension in the cord.

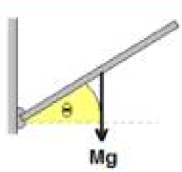


Example

A constant external torque of 50 Nm is applied to a wheel for 12 s, giving the wheel an angular velocity of 600 rev/min. Find the moment of inertia of the wheel.

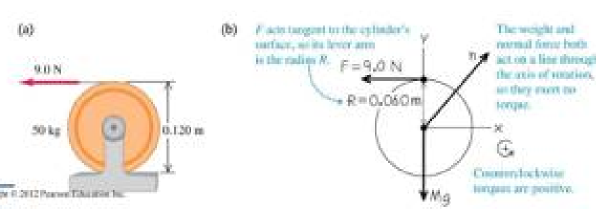
Example

A 12-kg rod of length 0.8-m is hinged at one end. Initially it is kept at θ= 40⁰ to above the horizontal. What is the initial acceleration of the rod ?

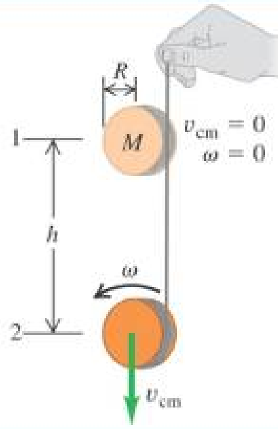


Torque and angular acceleration for a rigid body

We wrap a light cable around a solid cylinder of mass of 50 kg and diameter 0.12 m, which rotates about a horizontal axis.. We pull the free end of the cable with constant 9-N force. Find the cable's acceleration.

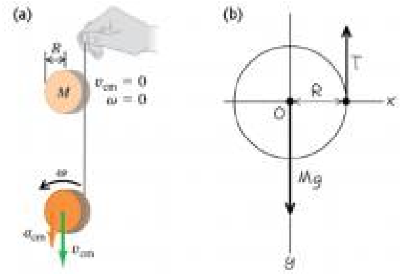


A yo-yo



Acceleration of a yo-yo

- We have translation and rotation, so we use Newton's second law for the acceleration of the center of mass and the rotational analog of Newton's second law for the angular acceleration about the center of mass.



Work in Rotational Motion

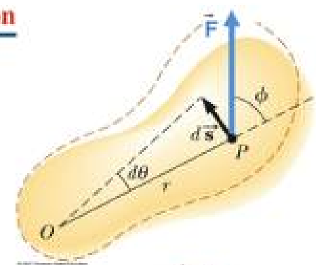
Find the work done by $\vec{F}$ on the object as it rotates through an infinitesimal distance $ds = r\,d\theta$

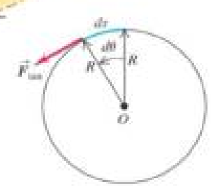
$$dW = \vec{F} \cdot d\vec{s} = (F \sin\phi) r\, d\theta$$
$$\tau = (F \sin\phi) r$$

$dW = \tau d\theta$ $\quad$ $W = \int \tau d\theta$

$$W = \tau \theta$$

The radial component of the force does no work because it is perpendicular to the displacement





Work-Kinetic Energy Theorem in Rotational Motion

Derivation:

$$\sum \tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\frac{d\theta}{dt} = I\frac{d\omega}{d\theta}\omega$$

$$dW = \sum \tau\, d\theta = I\omega\, d\omega$$

$$\sum W = \int_{\omega_i}^{\omega_f} I\omega\, d\omega = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

Power in Rotational Motion

The rate at which work is being done in a time interval dt is

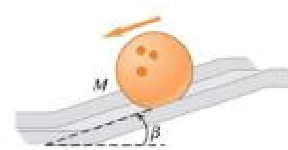
$$\text{Power} = \wp = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$$

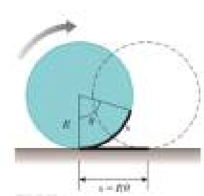
This is analogous to $\wp = Fv$ in a linear system

Pure Rolling Motion

In pure rolling motion, an object rolls without slipping

In such a case, there is a simple relationship between its rotational and translational motions





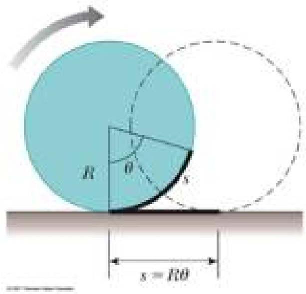
Rolling Object, Center of Mass

The velocity of the center of mass is

$$v_{CM} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

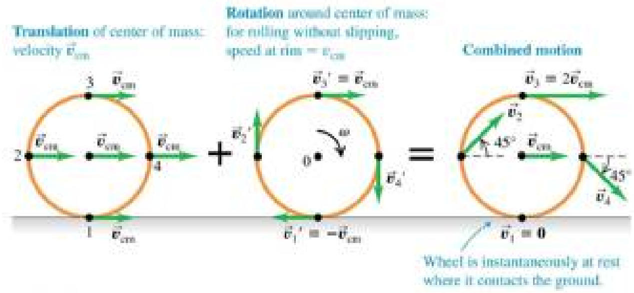
The acceleration of the center of mass is

$$a_{CM} = \frac{dv_{CM}}{dt} = R\frac{d\omega}{dt} = R\alpha$$



Rolling without slipping

- The condition for rolling without slipping is $v_{cm} = R\omega$.
- Figure 10.13 shows the combined motion of points on a rolling wheel.



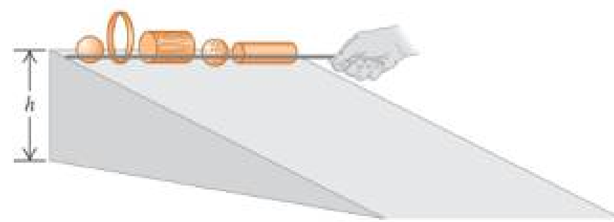
Total Kinetic Energy of a Rolling Object

The total kinetic energy of a rolling object is the sum of the translational energy of its center of mass and the rotational kinetic energy about its center of mass

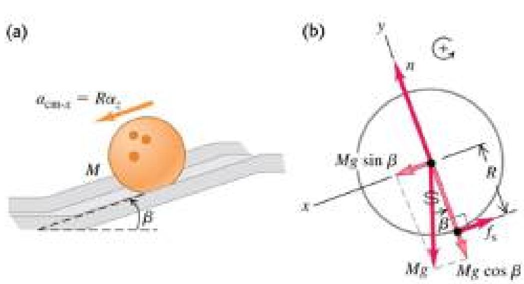
- $K = ½ I_{CM}\,\omega^2 + ½ Mv^2_{CM}$
  - The $½ I_{CM}\omega^2$ represents the rotational kinetic energy of the cylinder about its center of mass
  - The $½ Mv^2$ represents the translational kinetic energy of the cylinder about its center of mass

The race of the rolling bodies

- Follow Example 10.5 using Figure 10.16 below.



Acceleration of a rolling sphere

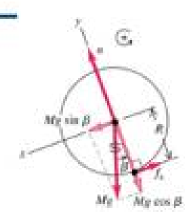


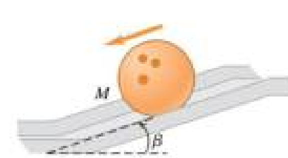
Total Kinetic Energy, Example 19

A solid ball of radius 12 cm starts from rest and rolls without slipping a distance x = 80 cm down a 25° incline.

A. Find the linear acceleration of the center of the ball.

B. Find the angular speed of the ball at the bottom of the incline.

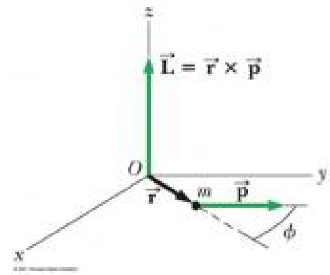




Angular Momentum of Particle

Consider a particle of mass m located at the vector position $\vec{\mathbf{r}}$ and moving with linear momentum $\vec{\mathbf{p}}$

The instantaneous angular momentum $\vec{\mathbf{L}}$ of a particle relative to the origin O is defined as the cross product of the particle's instantaneous position vector $\vec{\mathbf{r}}$ and its instantaneous linear momentum $\vec{\mathbf{p}}$



$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$$

More About Angular Momentum of Particle

The SI units of angular momentum are $(kg \cdot m^2)/s$

Both the magnitude and direction of the angular momentum depend on the choice of origin

The magnitude is $L = mvr \sin\theta$

- θ is the angle between v and r

The direction of $\vec{\mathbf{L}}$ is perpendicular to the plane formed by $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$

Angular Momentum of a Particle in Uniform Circular Motion

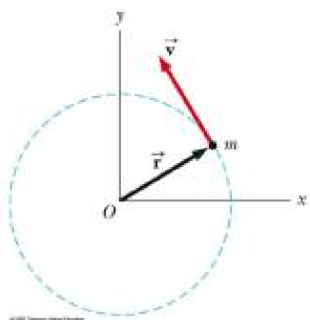
The vector $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$ is pointed out of the diagram

The magnitude is

$L = mvr \sin 90^{\circ} = mvr = mr^2\omega = I\,\omega$

- $\sin 90^{\circ}$ is used since v is perpendicular to r

A particle in uniform circular motion has a constant angular momentum about an axis through the center of its path



Torque and Angular Momentum

The torque is related to the angular momentum

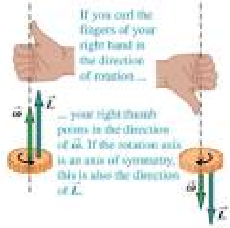
$$\Sigma\tau = I\alpha = I\frac{d\omega}{dt} = \frac{d\,I\omega}{dt} = \frac{d\mathbf{L}}{dt}$$

$$\sum \vec{\tau} = \frac{d\vec{\mathbf{L}}}{dt}$$

The torque acting on a particle is equal to the time rate of change of the particle's angular momentum

Angular momentum

- The angular momentum of a rigid body rotating about a symmetry axis is parallel to the angular velocity and is given by $\vec{L} = I\vec{\omega}$.
- For any system of particles $\sum\vec{\tau} = d\vec{L}/dt$.



Angular Momentum of a System of Particles

The total angular momentum of a system of particles is defined as the vector sum of the angular momenta of the individual particles

- $\vec{\mathbf{L}}_{tot} = \vec{\mathbf{L}}_1 + \vec{\mathbf{L}}_2 + \ldots + \vec{\mathbf{L}}_n = \sum_i \vec{\mathbf{L}}_i = \omega \sum_i I_i = I_{tot}\,\omega$

$$L_{tot} = I_{tot}\,\omega$$

Differentiating with respect to time

$$\frac{d\vec{\mathbf{L}}_{tot}}{dt} = \sum_i \frac{d\vec{\mathbf{L}}_i}{dt} = \sum_i \vec{\tau}_i$$

Conservation of Angular Momentum

If the resultant external torque acting on the system is zero the total angular momentum of a system is constant in both magnitude and direction as

$$\frac{d\vec{L}_{tot}}{dt} = \sum_i \frac{d\vec{L}_i}{dt} = \sum_i \vec{\tau}_i$$

- Net torque = 0 -> means that the system is isolated

$$\vec{L}_{tot} = \text{constant or } \vec{L}_i = \vec{L}_f$$

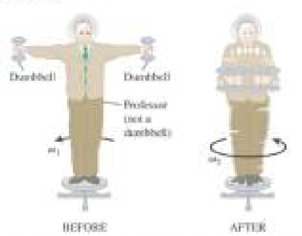
Conservation of Angular Momentum, cont

If the mass of an isolated system undergoes redistribution, the moment of inertia changes

- The conservation of angular momentum requires a compensating change in the angular velocity
- $I_i \omega_i = I_f \omega_f$ = constant
  - This holds for rotation about a fixed axis and for rotation about an axis through the center of mass of a moving system
  - The net torque must be zero in any case

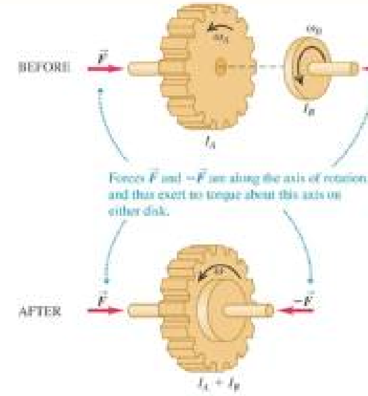
Conservation of angular momentum

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0-kg dumbbell in each hand He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells in to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg · m^2 with arms outstretched and 2.2 kg · m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.



A rotational "collision"

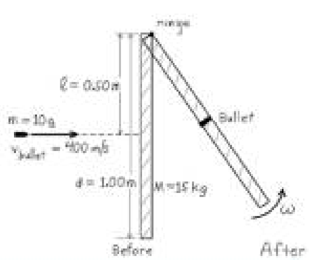
- Derive an expression for ω.



Angular momentum in a crime bust

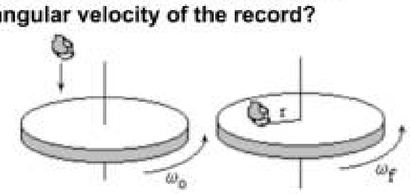
- A bullet hits a door causing it to swing.

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed.



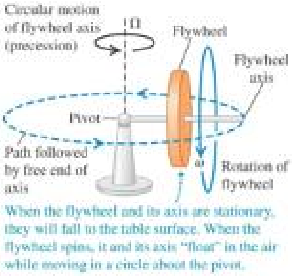
Example

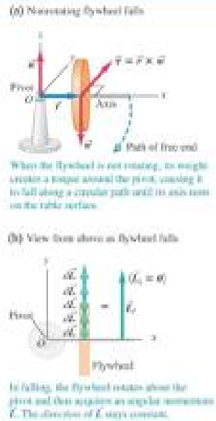
A 12-g object is dropped onto a record of rotational inertia I = 2x10^{-4} kgm^2 initially rotating freely at 33 revolutions per minute. The object adheres to the surface of the record at distance 8 cm from its center. What is the final angular velocity of the record?



Gyroscopes and precession

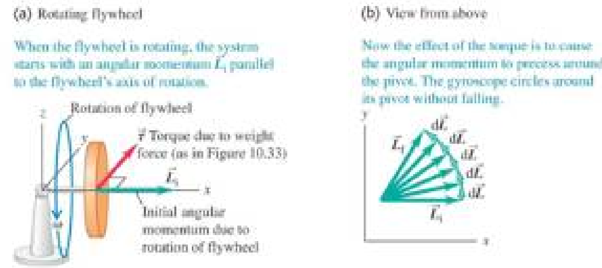
- For a gyroscope, the axis of rotation changes direction. The motion of this axis is called *precession*.





A rotating flywheel

- Figure 10.34 below shows a spinning flywheel. The magnitude of the angular momentum stays the same, but its direction changes continuously.



A precessing gyroscopic

- Follow Example 10.13 using Figure 10.36.