Problem 12.11 A record turntable of radius R rotates at angular velocity ω (Fig. 12.15). The circumference is presumably Lorentz-contracted, but the radius (being perpendicular to the velocity) is *not*. What's the ratio of the circumference to the diameter, in terms of ω and R? According to the rules of ordinary geometry, that has to be π. What's going on here? [This is known as **Ehrenfest's paradox**; for discussion and references see H. Arzelies, *Relativistic Kinematics*, Chap. IX (Elmsford, NY: Pergamon Press, 1966) and T. A. Weber, *Am. J. Phys.* **65**, 486 (1997).]

Problem 12.20

(a) Event A happens at point ($x_A = 5, y_A = 3, z_A = 0$) and at time t_A given by $ct_A = 15$; event B occurs at (10, 8, 0) and $ct_B = 5$, both in system $\mathcal{S}$.

(i) What is the invariant interval between A and B?

(ii) Is there an inertial system in which they occur *simultaneously*? If so, find its velocity (magnitude and direction) relative to $\mathcal{S}$.

(iii) Is there an inertial system in which they occur at the same point? If so, find its velocity relative to $\mathcal{S}$

(b) Repeat part (a) for $A = (2, 0, 0)$, $ct = 1$; and $B = (5, 0, 0)$, $ct = 3$.

Problem 12.22

(a) Draw a space-time diagram representing a game of catch (or a conversation) between two people at rest, 10 ft apart. How is it possible for them to communicate, given that their separation is spacelike?

(b) There's an old limerick that runs as follows:

> There once was a girl named Ms. Bright,
> Who could travel much faster than light.
> She departed one day,
> The Einsteinian way,
> And returned on the previous night.

What do you think? Even if she *could* travel faster than the speed of light, could she return before she set out? Could she arrive at some intermediate destination before she set out? Draw a space-time diagram representing this trip.

Problem 12.36 In classical mechanics Newton's law can be written in the more familiar form $\mathbf{F} = m\mathbf{a}$. The relativistic equation, $\mathbf{F} = d\mathbf{p}/dt$, *cannot* be so simply expressed. Show, rather, that

$$\mathbf{F} = \frac{m}{\sqrt{1 - u^2/c^2}} \left[\mathbf{a} + \frac{\mathbf{u}(\mathbf{u} \cdot \mathbf{a})}{c^2 - u^2} \right], \tag{12.73}$$

where $\mathbf{a} \equiv d\mathbf{u}/dt$ is the **ordinary acceleration.**

Problem 12.38 Define **proper acceleration** in the obvious way:

$$\alpha^\mu \equiv \frac{d\eta^\mu}{d\tau} = \frac{d^2 x^\mu}{d\tau^2}. \tag{12.74}$$

(a) Find α^0 and $\boldsymbol{\alpha}$ in terms of $\mathbf{u}$ and $\mathbf{a}$ (the ordinary acceleration).

(b) Express $\alpha_\mu \alpha^\mu$ in terms of $\mathbf{u}$ and $\mathbf{a}$.

(c) Show that $\eta^\mu \alpha_\mu = 0$.

(d) Write the Minkowski version of Newton's second law, Eq. 12.70, in terms of α^μ. Evaluate the invariant product $K^\mu \eta_\mu$.

Problem 12.66 "Derive" the Lorentz force law, as follows: Let charge q be at rest in $\bar{\mathcal{S}}$, so $\bar{\mathbf{F}} = q\bar{\mathbf{E}}$, and let $\bar{\mathcal{S}}$ move with velocity $\mathbf{v} = v\,\hat{\mathbf{x}}$ with respect to $\mathcal{S}$. Use the transformation rules (Eqs. 12.68 and 12.108) to rewrite $\bar{\mathbf{F}}$ in terms of $\mathbf{F}$, and $\bar{\mathbf{E}}$ in terms of $\mathbf{E}$ and $\mathbf{B}$. From these deduce the formula for $\mathbf{F}$ in terms of $\mathbf{E}$ and $\mathbf{B}$.

Problem 12.71 Generalize the laws of relativistic electrodynamics (Eqs. 12.126 and 12.127) to include magnetic charge. [Refer to Sect. 7.3.4.]