Problem 10.8 Confirm that the retarded potentials satisfy the Lorentz gauge condition. [*Hint:* First show that

$$\nabla \cdot \left(\frac{\mathbf{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla \cdot \mathbf{J}) + \frac{1}{\mathscr{r}}(\nabla' \cdot \mathbf{J}) - \nabla' \cdot \left(\frac{\mathbf{J}}{\mathscr{r}}\right),$$

where ∇ denotes derivatives with respect to $\mathbf{r}$, and ∇' denotes derivatives with respect to $\mathbf{r}'$. Next, noting that $\mathbf{J}(\mathbf{r}', t - \mathscr{r}/c)$ depends on $\mathbf{r}'$ both explicitly and through $\mathscr{r}$, whereas it depends on $\mathbf{r}$ only through $\mathscr{r}$, confirm that

$$\nabla \cdot \mathbf{J} = -\frac{1}{c}\dot{\mathbf{J}} \cdot (\nabla \mathscr{r}), \quad \nabla' \cdot \mathbf{J} = -\dot{\rho} - \frac{1}{c}\dot{\mathbf{J}} \cdot (\nabla' \mathscr{r}).$$

Use this to calculate the divergence of $\mathbf{A}$ (Eq. 10.19).]

Problem 10.15 I showed that *at most one* point on the particle trajectory communicates with $\mathbf{r}$ at any given time. In some cases there may be *no* such point (an observer at $\mathbf{r}$ would not see the particle—in the colorful language of General Relativity it is "beyond the **horizon**"). As an example, consider a particle in **hyperbolic motion** along the x axis:

$$\mathbf{w}(t) = \sqrt{b^2 + (ct)^2}\,\hat{\mathbf{x}} \qquad (-\infty < t < \infty). \tag{10.45}$$

(In Special Relativity this is the trajectory of a particle subject to a constant force $F = mc^2/b$.) Sketch the graph of w versus t. At four or five representative points on the curve, draw the trajectory of a light signal emitted by the particle at that point—both in the plus x direction and in the minus x direction. What region on your graph corresponds to points and times (x, t) from which the particle cannot be seen? At what time does someone at point x first see the particle? (Prior to this the potential at x is evidently zero.) Is it possible for a particle, once seen, to *disappear* from view?

Problem 10.22 Figure 2.35 summarizes the laws of *electrostatics* in a "triangle diagram" relating the *source* (ρ), the *field* ($\mathbf{E}$), and the *potential* (V). Figure 5.48 does the same for *magnetostatics*, where the source is $\mathbf{J}$, the field is $\mathbf{B}$, and the potential is $\mathbf{A}$. Construct the analogous diagram for *electrodynamics*, with sources ρ and $\mathbf{J}$ (constrained by the continuity equation), fields $\mathbf{E}$ and $\mathbf{B}$, and potentials V and $\mathbf{A}$ (constrained by the Lorentz gauge condition). Do not include formulas for V and $\mathbf{A}$ in terms of $\mathbf{E}$ and $\mathbf{B}$.

Problem 11.14 In Bohr's theory of hydrogen, the electron in its ground state was supposed to travel in a circle of radius 5×10^{-11}m, held in orbit by the Coulomb attraction of the proton.

According to classical electrodynamics, this electron should radiate, and hence spiral in to the nucleus. Show that $v \ll c$ for most of the trip (so you can use the Larmor formula), and calculate the lifespan of Bohr's atom. (Assume each revolution is essentially circular.)

Problem 11.22 A radio tower rises to height h above flat horizontal ground. At the top is a magnetic dipole antenna, of radius b, with its axis vertical. FM station KRUD broadcasts from this antenna at angular frequency ω, with a total radiated power P (that's averaged, of course, over a full cycle). Neighbors have complained about problems they attribute to excessive radiation from the tower—interference with their stereo systems, mechanical garage doors opening and closing mysteriously, and a variety of suspicious medical problems. But the city engineer who measured the radiation level at the base of the tower found it to be well below the accepted standard. You have been hired by the Neighborhood Association to assess the engineer's report.

(a) In terms of the variables given (not all of which may be relevant, of course), find the formula for the intensity of the radiation at ground level, a distance R from the base of the tower. You may assume that $a \ll c/\omega \ll h$. [*Note:* we are interested only in the *magnitude* of the radiation, not in its *direction*—when measurements are taken the detector will be aimed directly at the antenna.]

(b) How far from the base of the tower *should* the engineer have made the measurement? What is the formula for the intensity at this location?

(c) KRUD's actual power output is 35 kilowatts, its frequency is 90 MHz, the antenna's radius is 6 cm, and the height of the tower is 200 m. The city's radio-emission limit is 200 microwatts/cm^2. Is KRUD in compliance?

Problem 11.23 As you know, the magnetic north pole of the earth does not coincide with the geographic north pole—in fact, it's off by about 11°. Relative to the fixed axis of rotation, therefore, the magnetic dipole moment vector of the earth is changing with time, and the earth must be giving off magnetic dipole radiation.

(a) Find the formula for the total power radiated, in terms of the following parameters: Ψ (the angle between the geographic and magnetic north poles), M (the magnitude of the earth's magnetic dipole moment), and ω (the angular velocity of rotation of the earth). [*Hint:* refer to Prob. 11.4 or Prob. 11.12.]

(b) Using the fact that the earth's magnetic field is about half a gauss at the equator, estimate the magnetic dipole moment M of the earth.

(c) Find the power radiated. [*Answer:* 4×10^{-5} W]

(d) Pulsars are thought to be rotating neutron stars, with a typical radius of 10 km, a rotational period of $10^{-3}s$, and a surface magnetic field of 10^8 T. What sort of radiated power would you expect from such a star? [See J. P. Ostriker and J. E. Gunn, *Astrophys. J.* **157**, 1395 (1969).] [*Answer:* 2×10^{36} W]